Modelling distribution functions and fragmentation functions

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Abstract

We present examples for the calculation of the distribution and fragmentation functions using the representation in terms of non-local matrix elements of quark field operators. As specific examples, we use a simple spectator model to estimate the leading twist quark distribution functions and the fragmentation functions for a quark into a nucleon or a pion.

1 Introduction

From the theoretical point of view, the distribution functions are quark correlation functions involving matrix elements of bilocal combinations of quark fields [1]. In fact, using the formalism of light-cone quantization, we can give nice physical interpretations for the leading twist functions $f_1(x)$, $g_1(x)$ and $h_1(x)$. The first one represents the probability of finding a quark with (light-cone) momentum fraction x. The second one measures the probability of finding right-handed quarks with momentum fraction x minus the probability of finding left-handed quarks with the same momentum fraction. It is called a chirality distribution or (if we work in the infinite momentum frame) helicity distribution. In this talk we shall not consider the third one. A similar interpretation can be given for the fragmentation functions $D_1(z)$ and $G_1(z)$.

The stucture functions that appear in the cross sections for hard scattering processes are weighted sums of these (and other) distribution functions. However, due to the non-perturbative nature of QCD at low energies, we are not able to calculate from first principles the distribution functions nor the fragmentation functions of quarks into hadrons.

We are then lead to build models for the quark correlation functions. In this work we use a spectator model with some particular vertices that describe the interaction between the struck quark, the spectactor and the hadron. Our purpose is to calculate the leading twist distribution functions and fragmentation functions for the nucleon and the pion.

We are going to neglect the contributions of gluons. They lead to α_s and $\alpha_s \log Q^2$ corrections in the structure functions, the latter usually being absorbed in a scale dependence of the distribution and fragmentation functions. Thus, no QCD corrections will be computed. In this approximation the structure functions, the distributions and the fragmentation functions do not depend on Q^2 (for fixed $x \equiv -q^+/P^+$ or $z \equiv P_h^-/k^-$).

2 Distribution Functions

It is well known that the cross-section for an inclusive deep inelastic lepton-hadron scattering can be written in terms of the hadronic tensor

$$W_{\mu\nu} = \frac{1}{4M} \int \frac{d^4\xi}{2\pi} e^{iq\xi} \langle P,S|[J_\mu(\xi), J_\nu(0)]|P,S\rangle, \tag{1}$$

where M is the mass of the hadron, P^μ is its four-momentum, S^μ is its spin, q^μ is the momentum carried by the virtual photon and J_μ is the electomagnetic current. Since the hadron wave function is not known, we cannot calculate exactly the matrix element in (1). However, $W_{\mu\nu}$ admits the standard parametrization in terms of the dimensionless structure functions F_1, F_2, G_1 and G_2 (see, for instance, ref. [2]).

The leading contribution in $W_{\mu\nu}$ can be calculated from the well known handbag diagram. This diagram is composed of a hard scattering part (that can be calculated with perturbative QCD) and a soft hadronic part which must be modelled, since it involves unknown physics. This is the quark correlation function

$$\begin{aligned} \Phi_{ij}(k,P,S) &= \int \frac{d^4\xi}{(2\pi)^4}\; e^{ik\cdot\xi}\; \langle P,S \mid \bar\psi_j(0)\; \psi_i(\xi) \mid P,S\rangle \\ &= \frac{1}{(2\pi)^3} \sum_X \langle P,S \mid \bar\psi_j(0) \mid X\rangle\; \theta(P_X^0)\; \delta\left((k-P)^2 - M_X^2\right)\; \langle X \mid \psi_i(0) \mid P,S\rangle, \end{aligned} \tag{2}$$

where $\{X\}$ is a complete set of intermediate states. A similar definition can be given for the antiquark correlation function Φ^c.

Contained in this correlation function, is a set of distribution functions defined by

$$\Phi^{[\Gamma]}(x) \equiv \frac{1}{2} \int dk^- d\mathbf{k}_T\; Tr(\Phi\Gamma)\Big|_{k^+=xP^+} \tag{3}$$

where Γ is any 4×4 matrix. The leading twist distributions are

$$f_1(x) \;\equiv\; \Phi^{[\gamma^+]}(x) = \frac{1}{2} \int dk^- d\mathbf{k}_T\; Tr(\Phi\;\gamma^+) \tag{4}$$

$$g_1(x) \;\equiv\; \Phi^{[\gamma^+\gamma_5]}(x) = \frac{1}{2} \int dk^- d\mathbf{k}_T\; Tr(\Phi\;\gamma^+\gamma_5) \tag{5}$$

and the structure functions are given by

$$F_2(x)/x \;=\; 2F_1(x) = \sum_f Q_f^2\; \left[\Phi^{[\gamma^+]}(x) + \Phi^{c[\gamma^+]}(x)\right] \tag{6}$$

$$2\lambda\; G_1(x) \;=\; \sum_f Q_f^2\; \left[\Phi^{[\gamma^+\gamma_5]}(x) + \Phi^{c[\gamma^+\gamma_5]}(x)\right], \tag{7}$$

λ being the helicity of the hadron. Moreover, it can be shown that G_2 enters in the cross section multiplied by a factor $1/Q$ (it is a twist 3 structure function) and then we shall neglect it.

The most general expression for Φ is [3]

$$
\begin{aligned}
\Phi(k,P,S) \;=\;& M\ A_1 + A_2 \not{P} + A_3\ \not{k} + \frac{A_4}{M}\ \sigma^{\mu\nu} P_\mu k_\nu + i\ A_5\ k\cdot S\ \gamma_5 + M\ A_6\ \not{S}\ \gamma_5 \\
&+ \frac{A_7}{M}\ k\cdot S\ \not{P}\ \gamma_5 + \frac{A_8}{M}\ k\cdot S\ \not{k}\ \gamma_5 + \ i\ A_9\ \sigma^{\mu\nu}\ \gamma_5\ S_\mu P_\nu \\
&+ i\ A_{10}\ \sigma^{\mu\nu}\ \gamma_5\ S_\mu k_\nu + \ i\ \frac{A_{11}}{M^2}\ k\cdot S\ \sigma^{\mu\nu}\ \gamma_5\ k_\mu P_\nu + \frac{A_{12}}{M}\ \epsilon_{\mu\nu\rho\sigma}\ \gamma^\mu P^\nu k^\rho S^\sigma.
\end{aligned}
\tag{8}
$$

Hermiticity requires that all the amplitudes $A_i(k\cdot P, k^2)$ are real. When time-reversal invariance applies, the amplitudes A_4, A_6 and A_{12} vanish.

The distributions can be written in terms of these amplitudes [4]:

$$
f_1(x) \;=\; M^4\ \pi \int d\sigma d\tau\ \theta(x\sigma - \tau - x^2)\ (A_2 + xA_3) \tag{9}
$$

$$
g_1(x) \;=\; M^4\ \pi \int d\sigma d\tau\ \theta(x\sigma - \tau - x^2) \left[-A_6 - \left(\frac{\sigma}{2} - x \right)\ (A_7 + xA_8) \right] \tag{10}
$$

with $\tau = k^2/M^2, \sigma = 2k\cdot P/M^2$.

3 Fragmentation Functions

Also the calculation of the fragmentation functions involves a soft hadronic part given by

$$
\Delta_{ij}(k,P_h,S_h) = \sum_X \int \frac{d^4\xi}{(2\pi)^4}\ e^{ik\xi}\ \langle 0|\psi_i(\xi)|P_h,S_h,X\rangle\ \langle X,P_h,S_h|\bar{\psi}_j(0)|0\rangle. \tag{11}
$$

This function admits an expansion similar to (8) but we are not going to need it. Instead, we define immediately the set of functions

$$
\Delta^{[\Gamma]}(z) \equiv \frac{1}{4z} \int dk^+ d^2\mathbf{P}_{hT}\ Tr(\Delta\Gamma)\Big|_{k^- = P_h^-/z} \tag{12}
$$

where z is the (light-cone) momentum fraction carried by the observed hadron originated by the fragmentation of the quark and $\mathbf{P}_{hT}$ is the transverse momentum of this hadron in a frame where the hadron parent has no transverse momentum.

The leading twist fragmentation functions are

$$
D_1(z) \;\equiv\; \Delta^{[\gamma^-]}(z) \tag{13}
$$

$$
G_1(z) \;\equiv\; \Delta^{[\gamma^-\gamma_5]}(z) \tag{14}
$$

and we can easily derive a useful relation between these fragmentation functions and the previously defined distribution functions:

$$
D_1(z) \;=\; z/2\ f_1(1/z) \tag{15}
$$

$$
G_1(z) \;=\; z/2\ g_1(1/z). \tag{16}
$$

4 The Spectator Model

In the spectator model the sum over all possible intermediate states in (2) is reduced to a single term, corresponding to the spectator $|X_{sp}\rangle$. It seems reasonable to consider this spectator as a (scalar or axial vector) diquark. The amplitude for obtaining the state $|X_{sp}\rangle$, after removing a quark from the nucleon is given by

$$\langle X_{sp} \mid \psi_i(0) \mid P, S\rangle = \left[\frac{i}{\not k - m + i\epsilon} \, \Gamma^{s,a} \; u(P,S) \right]_i . \tag{17}$$

We are going to assume that the vertices have the following structure:

$$\begin{aligned} \Gamma^s &= g_s(k^2) \; I & (18) \\ \Gamma^a_\alpha &= \frac{g_a(k^2)}{\sqrt{3}} \left(\gamma_\alpha \; \gamma_5 - \frac{P_\alpha}{M} \; \gamma_5 \right) . & (19) \end{aligned}$$

As form factors, we follow ref. [5] and choose

$$g_{s,a}(k^2) = N_{s,a} \left(M^2 \right)^{\alpha - 1} \frac{k^2 - m^2}{(k^2 - \Lambda^2)^\alpha}. \tag{20}$$

Throughout this section, m stands for the (effective) quark mass, Λ is a parameter to adjust and N is a normalization constant. The other parameter of the model is the mass of the diquark. In order to fit the experimental data as well as possible, we set $m = 0.36$ GeV, $m_s = 0.6$ GeV, $m_a = 0.8$ GeV and $\Lambda = 0.7$ GeV.

The choice $\alpha = 2$ in (20) ensures the correct behaviour of the distributions at large x. For the case of a scalar spectator the correlation function is then

$$\begin{aligned} \Phi^s = & \frac{\theta(P^0_s)}{16\pi^3} \; \frac{\delta((k-P)^2 - m_s^2)}{(k^2 - m^2)^2} \times \\ & [(\not k + m) \; \Gamma^s(k,P) \; (\not P + M) \; (1 + \gamma_5 \not S) \; \gamma_0 \; \Gamma^{s\dagger}(k,P) \; \gamma_0 \; (\not k + m)] \end{aligned} \tag{21}$$

where m_s is the mass of the scalar diquark. A similar expression is valid for the case of a vector diquark.

Using equations (17) - (21) we can readily compute the amplitudes A_i of (8) and then calculate the distributions using (9) and (10) and the fragmentation functions using (15) and (16). A calculation of fragmentation functions for baryons has been given in Ref. [6].

For the pion one can also employ the spectator model. The spectator is now an antiquark. The correlation function can be written in terms of only four amplitudes (because the pion is a spinless particle). The vertex is now

$$\Gamma^\pi = \frac{g(k^2)}{\sqrt{2}} \; \gamma_5 \; (\not P + M) \tag{22}$$

with the same form factor as in (20) but with $\alpha = 3/2$ and $\Lambda = 0.4$ GeV.

The preliminary results obtained with this model are shown in Fig. 3. At this moment no comparison with experiment is possible. This requires QCD evolution of these functions, what we postpone for a later work. At this stage the results ilustrate how the framework of quark correlation functions enables model calculations for distribution *and* fragmentation functions.

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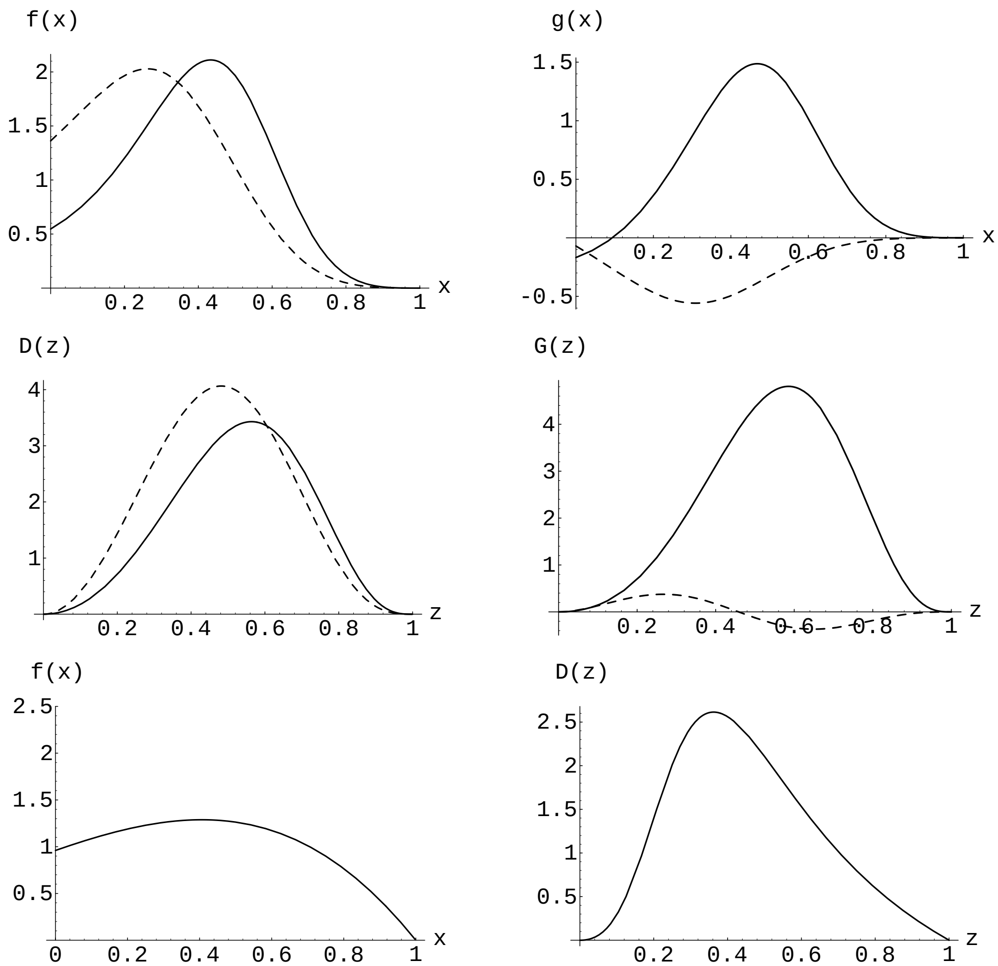

Figure 1: Nucleon distributions $f_1(x)$ and $g_1(x)$ (top); nucleon fragmentation functions $D_1(z)$ and $G_1(z)$ (middle); pion distribution function $f_1(x)$ (bottom-left); pion fragmentation function $D_1(z)$ (bottom-right). For the nucleon, the full line is for the scalar diquark and the dashed line for the vector diquark. The normalization of the fragmentation functions is arbitrary.

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